

assignment #7 (winter term 2005)
solutions will be presented Tuesday, 13-Dec-2005, 2 PM, o27/2203
<http://www.informatik.uni-ulm.de/pm/index.php?id=112>

Exercise 1. The addition of natural numbers written in successor notation (the natural number 3 is written as $s(s(s(0)))$) can be implemented by:

```
add(0,X,X).  
add(s(X),Y,s(Z)) :- add(X,Y,Z).
```

Write a constraintsolver in CHR, that solves as many as possible single add-constraints, e.g., the first Prolog clause implies:

```
add(0,Y,Z) <=> Y=Z.  
add(X,Y,Y) <=> X=0.
```

Exercise 2. Use the CHR Constraint `leq/2` from assignment #6-3 for a “constrain-and-generate” version of the sorting example from assignment #4-3. To this end, replace the `clpq`-Constraint `=<` by the CHR-constraint `leq`.

Your tests should (at least) include the following queries

```
?- permsortCHR([1,A,3],[1,3,7]).  
?- permsortCHR([2,A],X).  
?- permsortCHR([A,B,A],X).  
?- permsortCHR(List,[1,X,3]).  
?- permsortCHR([1,X,Y],[X,1,Y]), permsortCHR([4,5,10],[Z,Y,W]).
```

Exercise 3. Write a CHR program for the `maximum(X,Y,Z)`-constraint, which succeeds iff Z is the maximum of X and Y . Use your implementation of the `leq/2`- and `less/2`-constraints from assignment #5-3. Consider the following items in this order:

- a) Write a CHR rule, which computes the maximum Z of two given numbers X and Y .
- b) Enhance by inserting a CHR rule, such that queries like `?- maximum(X,X,3)` can be handled satisfactorily (we expect $X = 3$).
- c) Insert a CHR rule which, given the constraint `maximum(X,Y,Z)`, propagates the constraints `X leq Z` and `Y leq Z`.
Test your program with the query `?- maximum(A,B,C),maximum(C,A,B)`.
- d) The query `maximum(X,3,X)` is solved (using (a-c)) to the conjunction of `maximum(X,3,X)` and `3 leq X`.
Explain the connection between these two constraints.
- e) Insert CHR rules, such that under a given inequality between X and Y (e.g. `X leq Y`) the constraint `maximum(X,Y,Z)` is replaced by the implied equality (e.g. `Y = Z`).
- f) Explain, why the answer to the query `?- maximum(X,3,X)` (using (a-e)) is different from the one in d).
- g) Insert a CHR rule, such that for a query like `?- X less Z, maximum(X,Y,Z)` the answer `Y=Z` is returned.
- h) In order to handle queries like `?- maximum(X,Y,3),maximum(X,Y,5)` in a satisfactory manner, extend the program for a rule for this class of queries.
- i) Which are the sufficient conditions, given that two variables are unequal, to allow more inferences from `maximum(X,Y,Z)`.
Implement this case by (additional) rules in your CHR program.

Exercise 4. Sudoku (Number Place) is a logic-based placement puzzle on a 9×9 board: “Fill in the grid so that every row, every column, and every 3×3 box contains the digits 1 through 9.” [<http://www.sudoku.com>].

Write a `clpfd`-based Sudoku solver and run test cases for at least three instances of the puzzle. You can find (lots of) additional information on the web.