Excercise 1 (Top-Down Fibonacci). Consider the following two top-down implementations of Fibonacci numbers.

**Variant 1:**

```
fib(0, M) <=> M=1.
fib(1, M) <=> M=1.
fib(N, M) <=> N>=2 | N1 is N-1, N2 is N-2, fib(N1, M1), fib(N2, M2), M is M1+M2.
```

**Variant 2:**

```
fib(N, M1) \ fib(N, M2) <=> M1=M2.
fib(0, M) ==> M=1.
fib(1, M) ==> M=1.
fib(N, M) ==> N>=2 | N1 is N-1, N2 is N-2, fib(N1, M1), fib(N2, M2), M is M1+M2.
```

Predict the time complexity of each variant. Which variant is more efficient? Implement both variants and verify your prediction.

Excercise 2 (Bottom-Up Fibonacci). Implement a bottom-up implementation of Fibonacci numbers based on the following rule (which is non-terminating in the current form):

```
fib(N1, M1), fib(N2, M2) ==> N2=:=N1+1 | N3 is N2+1, M3 is M1+M2, fib(N3, M3).
```

How do you ensure termination?

Excercise 3 (Top-Down Factorial). Implement a CHR program computing the factorial of a natural number in a top-down manner.

Excercise 4 (Top-Down Binomial Coefficients). Binomial coefficients (Pascal’s triangle) can be computed using the following recursive formula:

\[
\binom{n}{k} := \begin{cases} 
1 & \text{if } k = 0 \text{ or } k = n \\
\binom{n-1}{k} + \binom{n-1}{k-1} & \text{if } 0 < k < n \\
0 & \text{otherwise}
\end{cases}
\]

Implement a CHR program to compute binomial coefficients efficiently in a top-down manner.

Excercise 5 (Bottom-Up Binomial Coefficients). Implement a CHR program to compute binomial coefficients in a bottom-up manner.
Excercise 6 (Fibonacci Approximation of the Golden Ratio). The so-called “golden ratio” (sectio aurea in latin) has been known for at least 2,400 years and among other properties has been widely considered as aesthetically especially pleasing. Mathematically, the golden ratio is an irrational constant which equals

$$\varphi = \frac{1 + \sqrt{5}}{2}$$

The golden ration is closely related to the Fibonacci sequence:

$$\lim_{n \to \infty} \frac{F(n + 1)}{F(n)} = \varphi$$

Use this relationship to implement a CHR program which computes an approximation to the golden ratio up to some user-defined accuracy.