CHR - a common platform for rule-based approaches
Renaissance of rule-based approaches

Results on rule-based system re-used and re-examined for

- Business rules and Workflow systems
- Semantic Web (e.g. validating forms, ontology reasoning, OWL)
- UML (e.g. OCL invariants) and extensions (e.g. ATL)
- Computational Biology
- Medical Diagnosis
- Software Verification and Security
Overview

Embedding rule-based approaches in CHR
Using source-to-source transformation (no interpreter, no compiler)

- Rewriting- and graph-based formalisms
  - Term Rewriting Systems
  - Chemical Abstract Machine and Multiset Transformation
  - Colored Petri Nets

- Rule-based systems
  - Production Rules
  - Event-Condition-Action Rules
  - Logical Algorithms

- Logic- and constraint-based programming languages
  - (Deductive Databases)
  - Prolog and Constraint Logic Programming
  - Concurrent Constraint Programming
Embeddings in CHR

Advantages

- **Advantages of CHR for execution**
  - Efficiency, also optimal complexity possible
  - Abstract execution by constraints, even when arguments unknown
  - Incremental, anytime, online algorithms for free
  - Concurrent, parallel for confluent programs

- **Advantages of CHR for analysis**
  - Decidable confluence and operational equivalence
  - Estimating complexity semi-automatically
  - Logic-based declarative semantics for correctness

- *Embedding allows for comparison and cross-fertilization (transfer of ideas)*
Potential shortcomings of embeddings in CHR

⇒ Use extensions of CHR (dynamic CHR covers all
  ► for built-in “negation” of rb systems, deductive db and Prolog
    ⇒ CHR with negation-as-absence
  ► for conflict resolution of rule-based systems
    ⇒ CHR with priorities
  ► for built-in search of Prolog, constraint logic programming
    ⇒ CHR with disjunction or search library
  ► for ignorance of duplicates of rule-based formalisms
    ⇒ CHR with set-based semantics
  ► for diagrammatic notation of graph-based systems
    ⇒ CHR with graphical interface

Instead of extensions, special-purpose CHR programs can be used.
Positive ground range-restricted CHR

- All approaches can be embedded into simple CHR fragment (except Prolog, constraint logic programming)
  - **ground**: queries ground
  - **positive**: no built-ins in body of rule
  - **range-restricted**: variables in guard and body also in head
- These conditions imply
  - Every state in a computation is ground
  - CHR constraints do not delay and wake up
  - Guard entailment check is just test
  - Computations cannot fail
- Conditions can be relaxed: auxiliary functions as non-failing built-ins in body
Distinguishing features of CHR for programming

Unique combination of features

- **Multiple Head Atoms** not in other programming languages
- **Propagation rules** only in production rules, deductive databases, Logical Algorithms
- **Constraints** only in constraint-based programming
  - Logical variables instead of ground representation
  - Constraints are reconsidered when new information arrives
  - Notion of failure due to built-in constraints
- **Logical Declarative Semantics** only in logic-based prog.
  - CHR computations justified by logic reading of program
Embedding fragments of CHR in other rule-based approaches

Possibilities are rather limited (without interpreter or compiler)

- **Positive ground range-restricted fragment** embeddable into
  - Rule-based systems with negation and Logical Algorithms
  - Only simplification rules in Rewriting- and Graph-based approaches (except Petri-nets)
  - Only propagation rules in deductive databases

- **Single-headed rules** embeddable into
  - Concurrent constraint programming languages
Rewriting-based and graph-based formalisms

Embedding of classical computational formalisms in CHR

- States mapped to CHR constraints
- Transitions mapped to CHR rules

Results in certain types of positive ground range-restricted CHR simplification rules (PGRS rules)
Rewriting-based and graph-based formalisms (I)

- **Term rewriting systems (TRS)**
  - Replace subterms given term according to rules until exhaustion
  - Analysis of TRS has inspired related results for CHR (termination, confluence)
  - Formally based on equational logic

- **Functional Programming (FP)**
  - Related to syntactic fragment of TRS extended with built-ins

- **Graph transformation systems (GTS)**
  - Generalise TRS: graphs are rewritten under matching morphism
Rewriting-based and graph-based formalisms (II)

- **GAMMA**
  - Based solely on multiset rewriting
  - Basis of Chemical Abstract Machine (CHAM)
  - Chemical metaphor of reacting molecules

- **Graph-based diagrammatic formalisms**
  - Examples: Petri nets, state charts, UML activity diagrams
  - Computation: tokens move along arcs
  - Token at nodes correspond to constraints, arcs to rules
Term rewriting systems (TRS) and CHR

Principles

- Rewriting rules: directed equations between ground terms
- Rule application: Given a term, replace subterms that match lhs. of rule with rhs. of rule
- Rewriting until no further rule application is possible

Comparison to CHR

- TRS locally rewrite subterms at fixed position in one ground term (functional notation)
- CHR globally manipulates several constraints in multisets of constraints (relational notation)
- TRS rules: **no built-ins**, no guards, no logical variables
- TRS rules: restrictions on occurrences of pattern variables

TRS map to subset of positive ground range-restricted simplification rules without built-ins over binary CHR constraint for equality
Flattening

Transformation forms basis for embedding TRS (and FP) in CHR
- Opposite of variable elimination, introduce new variables
- Flattening function transforms atomic equality constraint $\text{eq}$ between nested terms into conjunction of flat equations

Definition (Flattening function)

$$[X \text{ eq } T] := \begin{cases} X \text{ eq } T & \text{if } T \text{ is a variable} \\ X \text{ eq } f(X_1, \ldots, X_n) \land \bigwedge_{i=1}^{n}[X_i \text{ eq } T_i] & \text{if } T = f(T_1, \ldots, T_n) \end{cases}$$

($X$ variable, $T$ term, $X_1 \ldots X_n$ new variables)
Embedding TRS in CHR

Definition (Rule scheme for term rewriting rule)

TRS rule

\[ S \rightarrow T \]

translates to CHR simplification rule

\[ [X \text{ eq } S] \iff [X \text{ eq } T] \]

\(X\) new variable, \(\text{eq}\) CHR constraint
Example (Addition of natural numbers)

Example (TRS)

\[
\begin{align*}
0 + Y & \rightarrow Y. \\
s(X) + Y & \rightarrow s(X+Y).
\end{align*}
\]

Example (CHR)

\[
\begin{align*}
T & \equiv T1 + T2, \ T1 \equiv 0, \ T2 \equiv Y \iff T \equiv Y. \\
T & \equiv T1 + T2, \ T1 \equiv s(T3), \ T3 \equiv X, \ T2 \equiv Y \iff \\
& \quad T \equiv s(T4), \ T4 \equiv T5 + T6, \ T5 \equiv X, \ T6 \equiv Y.
\end{align*}
\]
Example (Logical conjunction)

**Example (TRS)**

- \text{and}(0, Y) \rightarrow 0.
- \text{and}(X, 0) \rightarrow 0.
- \text{and}(1, Y) \rightarrow Y.
- \text{and}(X, 1) \rightarrow X.
- \text{and}(X, X) \rightarrow X.

**Example (CHR)**

- \text{T} \text{ eq and}(T1, T2), \ T1 \text{ eq 0, T2 eq Y} \iff \text{T eq 0}.
- \text{T} \text{ eq and}(T1, T2), \ T1 \text{ eq X, T2 eq 0} \iff \text{T eq 0}.
- \text{T} \text{ eq and}(T1, T2), \ T1 \text{ eq 1, T2 eq Y} \iff \text{T eq Y}.
- \text{T} \text{ eq and}(T1, T2), \ T1 \text{ eq X, T2 eq 1} \iff \text{T eq X}.
- \text{T} \text{ eq and}(T1, T2), \ T1 \text{ eq X, T2 eq X} \iff \text{T eq X}.
Completeness and nonlinearity

- **TRS linear** if variables occur at most once on lhs. and rhs.
- Translation by flattening incomplete if TRS nonlinear

**Example**

In the CHR translation, TRS rule $\text{and}(X, X) \rightarrow X$ applicable to $\text{and}(0, 0)$ but not directly to $\text{and}(\text{and}(0, 1), \text{and}(0, 1))$. 
Structure sharing (I)

- **Structure sharing** makes nonlinear but *confluent* TRS complete
- **Confluence**: Given term, each possible rule application sequence leads to same result

Implemented by simpagation rule enforcing functional dependency of `eq` (added at beginning of program)

**Definition (Rule for Structure Sharing)**

```
fd @ X eq T \ Y eq T <=> X=Y.
```

**Example**

```
Z eq and(X,Y), W eq and(X,Y)  
now reduces to  
Z eq and(X,Y), W=Z
```
Structure sharing (II)

- Rule \( \text{fd} \) removes equations \( \Rightarrow \) other rules may no longer apply
- Solution: Additional CHR rules, so that rules also apply after application of \( \text{fd} \) (regain confluence)
- Corresponds to enforcing set-based semantics as in LA
  - Transformation applies to CHR rules in general
  - Generation of new rule variants by unifying head constraints

**Example**

- TRS rule \( \text{and}(X, X) \rightarrow X \) translates to
  \[
  T \text{ eq } \text{and}(T_1, T_2), \ T_1 \text{ eq } X, \ T_2 \text{ eq } X \Leftrightarrow T \text{ eq } X
  \]
- Expects \( T_1 \text{ eq } X \) and \( T_2 \text{ eq } X \) even if \( T_1 = T_2 \); unify them:
- Additional rule \( T \text{ eq } \text{and}(T_1, T_1), \ T_1 \text{ eq } X \Leftrightarrow T \text{ eq } X \)
Functional programming (FP)

- FP can be seen as programming language based on TRS formalism
  - Extended by built-in functions and guard tests
  - Syntactic restrictions on lhs. of rewrite rule:
    Matching only at outermost redex of lhs
Translation

Definition (Rule scheme for functional program rule)

FP rewrite rule

\[ S \rightarrow G \mid T \]

translates to CHR simplification rule

\[ X \text{ eq } S \Leftrightarrow G \mid [X \text{ eq } T] \]

\((X \text{ new variable})\)
Additional generic rules for data and auxiliary functions

\[ X \text{ eq } T \Leftrightarrow \text{datum}(T) \mid X=T. \]
\[ X \text{ eq } T \Leftrightarrow \text{builtin}(T) \mid \text{call}(T,X). \]

\((\text{call}(T,X) \text{ calls built-in function } T, \text{ returns result in } X)\)

Generic rules can be applied at compile time to body (and head)
Examples (Adding natural numbers, logical conjunction)

Example (Addition of natural numbers in CHR)

\[
\begin{align*}
T \text{ eq } 0 \text{+} Y & \iff T \text{ eq } Y. \\
T \text{ eq } s(X) \text{+} Y & \iff T = s(T4), T4 \text{ eq } T5 + T6, T5 \text{ eq } X, T6 \text{ eq } Y.
\end{align*}
\]

Example (Logical conjunction in CHR)

\[
\begin{align*}
T \text{ eq } \text{and}(0, Y) & \iff T = 0. \\
T \text{ eq } \text{and}(X, 0) & \iff T = 0. \\
T \text{ eq } \text{and}(1, Y) & \iff T \text{ eq } Y. \\
T \text{ eq } \text{and}(X, 1) & \iff T \text{ eq } X. \\
T \text{ eq } \text{and}(X, Y) & \iff T \text{ eq } X.
\end{align*}
\]
Example (Fibonacci Numbers)

Example (Fibonacci in FP)

\[
\begin{align*}
\text{fib}(0) & \rightarrow 1. \\
\text{fib}(1) & \rightarrow 1. \\
\text{fib}(N) & \rightarrow N \geq 2 \mid \text{fib}(N-1) + \text{fib}(N-2).
\end{align*}
\]

Example (Fibonacci in CHR)

\[
\begin{align*}
T \equiv \text{fib}(0) & \iff T = 1. \\
T \equiv \text{fib}(1) & \iff T = 1. \\
T \equiv \text{fib}(N) & \iff N > 2 \mid \text{call}(F1+F2,T), \\
& \quad F1 \equiv \text{fib}(N1), \text{call}(N-1,N1), \\
& \quad F2 \equiv \text{fib}(N2), \text{call}(N-2,N2).
\end{align*}
\]

(Generic rules for datum and built-in already applied in bodies)
Graph transformation systems (*)

- Can be seen as nontrivial generalization of TRS
  - Instead of terms, graphs are rewritten under matching morphism
- Encoding of GTS production rules exists for CHR (complete, sound)
- Confluence: GTS joinability of critical pairs mapped to joinability of specific critical pairs in CHR
GAMMA

- Chemical metaphor: molecules in solution react according to reaction rules
- Reaction in parallel on disjoint sets of molecules
- Molecules modeled as unary CHR constraints, reactions as rules

**Definition (GAMMA)**

- GAMMA program: pairs \((c/n, f/n)\) (predicate \(c\), function \(f\))
- \(f\) applied to molecules for which \(c\) holds
- Result \(f(x_1, \ldots, x_n) = \{y_1, \ldots, y_m\}\) replaces \(\{x_1, \ldots, x_n\}\) in \(S\)
- Repeat until exhaustion
GAMMA Translation

**Definition (Rule scheme for GAMMA pair)**

A GAMMA pair \((c/n, f/n)\) translated to a simplification rule:

\[
d(x_1), \ldots, d(x_n) \Leftrightarrow c(x_1, \ldots, x_n) \mid f(x_1, \ldots, x_n),
\]

where \(f\) is defined by rules of the form:

\[
f(x_1, \ldots, x_n) \Leftrightarrow G \mid D, d(y_1), \ldots, d(y_m),
\]

\((d\) wraps molecules, \(c\) built-in, \(G\) guard, \(D\) auxiliary built-ins)\

Can unfold \(f\) if defined by one rule, optimize to simpagation rules

(CHR simplification rules can be translated to GAMMA)
GAMMA examples and translation into CHR

Example (Minimum)

\[
\begin{align*}
\text{min} &= (</2, \text{first}/2) \\
\text{min} &@ d(X), d(Y) \iff X < Y \mid \text{first}(X, Y).
\end{align*}
\]
\[
\begin{align*}
\text{first}(X, Y) &\iff d(X).
\end{align*}
\]

Example (Greatest Common Divisor)

\[
\begin{align*}
\text{gcd} &= (</2, \text{gcdsub}/2) \\
\text{gcd} &@ d(X), d(Y) \iff X < Y \mid \text{gcdsub}(X, Y).
\end{align*}
\]
\[
\begin{align*}
\text{gcdsub}(X, Y) &\iff d(X), d(Y-X).
\end{align*}
\]

Example (Prime sieve)

\[
\begin{align*}
\text{prime} &= (\text{div}/2, \text{first}/2) \\
\text{prime} &@ d(X), d(Y) \iff X \text{ div } Y \mid \text{first}(X, Y).
\end{align*}
\]
\[
\begin{align*}
\text{first}(X, Y) &\iff d(X).
\end{align*}
\]

Examples can be optimised into single simpagation rules.
Petri nets

- Petri nets consist of
  - Places (P) (○)
  - Tokens (●)
  - Arcs (→)
  - Transitions (T) (□)

- Tokens reside in places, move along arcs through transitions

- Transitions
  - Fire if tokens are present on all incoming arcs:
  - tokens removed from incoming arcs, placed on outgoing arcs
Example (Petri net)

- Places P1 - P4
  - P1 and P4 contain one token, P3 contains two tokens

- Transitions T1 and T2
  - T1 needs one incoming token, produces two outgoing tokens
  - T2 needs two incoming token, produces two outgoing tokens
Colored Petri nets

- Standard Petri nets translate to tiny fragment of CHR
  - Nullary constraints and simplification rules
- Colored Petri nets: tokens have different colors
  - Places allow only certain colors
  - Number of colors is fixed and finite
  - Transitions guarded with conditions on token colors
  - Equations at transitions generate new tokens
  - Sound and complete translation to CHR exists

(Colored) Petri Nets are not turing-complete.
Colored Petri nets Translation

Simplification rules over unary constraints

- Places $\rightarrow$ unary CHR constraint symbols
- Tokens $\rightarrow$ arguments of place constraints
- Colors $\rightarrow$ finite domains (possible values)
- Transitions $\rightarrow$ CHR simplification rules
  - Incoming arc annotation $\rightarrow$ rule head
  - Outgoing arc annotation $\rightarrow$ rule body
  - Transition guard $\rightarrow$ rule guard
  - Transition equation $\rightarrow$ rule body
Example – The Dining philosophers Problem

- The dining philosophers problem
  - Philosophers at round table, between each philosopher one fork
  - Philosophers either eat or think
  - For eating, forks from both sides required
  - After eating, philosophers start thinking again

- Dining philosophers as Colored Petri net
  - Philosopher, fork → colored tokens
  - Tokens $x, y$ are neighbors at round table if $y = (x + 1) \mod 3$
  - Places: eat $e$, think $t$, fork $f$
  - Arcs: eat-to-think $et$ and think-to-eat $te$
Three (3) dining philosophers Colored Petri net

\[
y = (x+1) \mod 3 : \text{true}
\]
Three dining philosophers CHR translation

Example (Dining philosophers in CHR)

\[
\begin{align*}
te @ & \ t(X), f(X), f(Y) \Leftrightarrow [X,Y]::[0,1,2], Y=(X+1) \mod 3 \mid e(X). \\
et @ & \ e(X) \Leftrightarrow X::[0,1,2] \mid Y=(X+1) \mod 3, t(X), f(X), f(Y).
\end{align*}
\]

- \( \forall : L \) - variable or variables in list take only values from list
- **Query**: \( t(0), t(1), t(2), f(0), f(1), f(2) \)
- **Note**: et rule is reverse of te rule (nonterminating)
- Observe loop: add e.g. \( e(X) \Rightarrow \text{println}(e(X)) \) in front
- Use conflict resolution to obtain fair rule scheduling
- Can be easily generalized to any given finite \( n \)
- CHR Rules for Colored Petri Nets are similar to rules for GAMMA (but only finite domains)
Rule-based systems

Overview

Use ground representation

- **Production rule systems**
  - First rule-based systems
  - Imperative, destructive assignment \(\Rightarrow\) no declarative semantics
  - Developed in the 1980s

- **Event-Condition-Action (ECA) rules**
  - Extension of production rules
  - For active database systems
  - Hot research topics in the mid-1990s
  - Some aspects standardized in SQL-3
Overview, contd.

- **Business rules**
  - Constrain structure and behavior of business
  - Describe operation of company and interaction with customers and other companies
  - Recent commercial approach (since end of 1990s)

- **Logical Algorithms formalism**
  - Hypothetical declarative production rule language
  - Similar to Deductive Databases
  - Overshadowing information instead of removal
  - More recent approach (early 2000s)
Production rule systems

Working memory stores facts (working memory elements, WME)
Facts have name and named attributes

Production rule

if \textit{Condition} then \textit{Action}

- \textbf{If}-clause: \textit{Condition}
  - Expression matchings describing facts

- \textbf{Then}-clause: \textit{Action}
  - insertion and removal of facts
  - IO statements
  - auxiliary functions
Production rule systems semantics

Execution cycle
1. Identify all rules with satisfied if-clause
2. **Conflict resolution** chooses one rule
   - e.g. based on priority
3. Then-clause is executed

Continue until exhaustion (all rules applied)
Embedding Production rules in CHR

- **Facts** translate to CHR constraints
  - Attribute name encoded by argument position
- **Production rules** translate to CHR *(generalised) simpagation rules*
  - If-clause forms head and guard, then-clause forms body
- Removal/insertion of facts by positioning in head/body of rule
- **Negation-as-absence** and conflict resolution implementable with *refined semantics* or CHR extensions
Translation

Definition (Rule scheme for production rule)

OPS5 production rule

\[(p \ N \ LHS \rightarrow RHS)\]

translates to CHR generalized simpagation rule

\[N \ @ \ LHS1 \ \backslash \ LHS2 \ \Leftrightarrow \ LHS3 \ \mid \ RHS'\]

**LHS** left hand side (if-clause), **RHS** right hand side (then-clause)

- **LHS1**: patterns of **LHS** for facts not modified in **RHS**
- **LHS2**: patterns of **LHS** for facts modified in **RHS**
- **LHS3**: conditions of **LHS**
- **RHS’**: **RHS** without removal (for **LHS2** facts)
Example (Fibonacci)

Example (OPS5)
(p next-fib (limit ^is <limit>))
{(fibonacci ^index {<i> <= <limit>}
  ^this-value <v1>
  ^last-value <v2>) <fib>}
--> (modify <fib> ^index (compute <i> + 1)
  ^this-value (compute <v1> + <v2>)
  ^last-value <v1>)
       (write (crlf) Fib <i> is <v1>))

Example (CHR)
next-fib @ limit(Lim), fibonacci(I,V1,V2) <=< I =< Lim |
       fibonacci(I+1,V1+V2,V1), write(fib I is V1), nl.

(Or attribute representation could be used e.g. fibonacci.index eq I)
Example (Greatest common divisor) (I)

**Example (OPS5)**

(p done-no-divisors
   (euclidean-pair ^first <first> ^second 1) -->
   (write GCD is 1) (halt) )

(p found-gcd
   (euclidean-pair ^first <first> ^second <first>) -->
   (write GCD is <first>) (halt) )

**Example (CHR)**

done-no-divisors @ euclidean_pair(First, 1) <= write(GCD is 1).

found-gcd @ euclidean_pair(First, First) <= write(GCD is First).
Example (Greatest common divisor) (II)

Example (OPS5)

\[
\text{(p switch-pair} \\
\quad \{(\text{euclidean-pair} \ ^\text{first} \ <\text{first}> \\
\quad \quad ^\text{second} \ {\ <\text{second}> \ > \ <\text{first}>}\} \ ) \\
\text{<e-pair>}\} \rightarrow \\
\text{(modify <e-pair>} \ ^\text{first} \ <\text{second}> \\
\quad \quad ^\text{second} \ <\text{first}>\) \\
\text{(write <first> -- <second> (crlf))} \ )
\]

Example (CHR)

\[
\text{switch-pair @ euclidean_pair(First, Second) } \leftrightarrow \text{ Second > First } \mid \\
\text{euclidean_pair(Second, First),} \\
\text{write(First--Second), nl.}
\]
Example (Greatest common divisor) (III)

**Example (OPS5)**

\[
(p \text{ reduce-pair} \\
\{(euclidean-pair ^\text{first} <\text{first}> \\
\hspace{1cm} ^\text{second} \{ <\text{second}> < <\text{first}> \} ) \\
<e\text{-pair}> \} \rightarrow \\
(modify <e\text{-pair}> ^\text{first} (\text{compute } <\text{first}>-<\text{second}>)) \\
(write <\text{first}> -- <\text{second}> (\text{crlf})) )
\]

**Example (CHR)**

\[
\text{reduce-pair } @ \text{ euclidean\_pair(First, Second)} \iff \text{Second < First} \\
\text{euclidean\_pair(First-Second, Second),} \\
\text{write(First--Second), nl.}
\]
Negation-as-absence

Negated pattern in production rules

- Satisfied if no fact satisfies condition
- Violates monotonicity (no more online, anytime, concurrency properties)

Example (Minimum in OPS5)

\[
(p \quad \text{minimum}
\quad (\text{num} \quad ^{\text{val}} \quad <x>)
\quad -(\text{num} \quad ^{\text{val}} < <x>)
\quad --> \quad (\text{make} \quad \text{min} \quad ^{\text{val}} <x>)
\)
Negation-as-absence II

Example (Transitive closure in OPS5)

\[(p \text{ init-path})\]
\[(\text{edge} ^\text{from} <x> ^\text{to} <y>)\]
\[-(\text{path} ^\text{from} <x> ^\text{to} <y>)\]
\[\rightarrow \text{ (make path} ^\text{from} <x> ^\text{to} <y>\text{)} \)\]

\[(p \text{ extend-path})\]
\[(\text{edge} ^\text{from} <x> ^\text{to} <y>)\]
\[(\text{path} ^\text{from} <y> ^\text{to} <z>)\]
\[\rightarrow \text{ (make path} ^\text{from} <x> ^\text{to} <z>\text{)} \)\]
Default reasoning

- Negation-as-absence can be used for default reasoning
  - Default is assumed unless contrary proven

Example (Marital status in OPS5)

```
(p default
  (person ^name <x>)
  -(married ^name <x>)
  -->
  (make single ^name <x>) )
```

Status single is default
But what happens if somebody marries?
What happens if somebody divorces?
Translation of Negation-as-absence

- Two approaches and one special case
  - Built-in constraints in guard (low-level)
  - CHR constraint in head (general approach)
  - Special case: body in head (changes semantics)
- Yet another approach: use explicit deletion of ECA rules (cleaner)
- Assume w.l.o.g. one negation per rule (not nested)
- Positive rule parts translated as before

Do it for generic CHR simpagation rules
CHR rules with negation-as-absence

Definition (Rule scheme for CHR rule with negation in head)

CHR generalised simpagation rule

\[ N \@ \text{LHS1} \setminus \text{LHS2} \rightarrowtext{(\text{NEG1,NEG2})} \iff \text{LHS3} \mid \text{RHS} \]

translates to CHR rules

\[ N1 \@ \text{LHS1} \land \text{LHS2} \Rightarrow \text{LHS3} \mid \text{check(\text{LHS1,LHS2})} \]
\[ N2 \@ \text{NEG1} \setminus \text{check\text{(LHS1,LHS2)}} \iff \text{NEG2} \mid \text{true} \]
\[ N3 \@ \text{LHS1} \setminus \text{LHS2} \land \text{check(\text{LHS1,LHS2)}} \iff \text{RHS} \]

- \( \text{NEG1} \) CHR constraints, \( \text{NEG2} \) built-in constraints
- \text{check}: auxiliary CHR constraint (different for each rule N)
- refined semantics ensures rule \( N2 \) is tried before rule \( N3 \)
- may not work incrementally when \( \text{NEG1} \) removed later
CHR constraint in head (II)

### Explanation

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1 @</td>
<td>( LHS1 \land LHS2 \Rightarrow LHS3 \mid \text{check}(LHS1, LHS2) )</td>
</tr>
<tr>
<td>N2 @</td>
<td>( \neg \text{check}(LHS1, LHS2) \Leftrightarrow \neg \text{NEG2} \mid \text{true} )</td>
</tr>
<tr>
<td>N3 @</td>
<td>( LHS1 \setminus LHS2 \land \text{check}(LHS1, LHS2) \Leftrightarrow RHS' )</td>
</tr>
</tbody>
</table>

- Given \( LHS \), check for absence of \( \neg \text{NEG} \) with \( \text{check} \) using \( N1 \)
- If \( \neg \text{NEG} \) found using \( N2 \), then remove \( \text{check} \)
- **Otherwise** apply rule using \( N3 \) and remove \( \text{check} \)
- Relies on rule order between \( N2 \) and \( N3 \)
- Works under refined semantics or with rule priorities
Examples

Example (Minimum in CHR)

\[
\text{num}(X) \implies \text{check}(\text{num}(X)).
\]
\[
\text{num}(Y) \setminus \text{check}(\text{num}(X)) \iff Y < X \mid \text{true}.
\]
\[
\text{num}(X) \setminus \text{check}(\text{num}(X)) \iff \text{min}(X).
\]

Example (Transitive closure in CHR, init-path rule)

\[
\text{e}(X, Y) \implies \text{check}(\text{e}(X, Y)).
\]
\[
\text{p}(X, Y) \setminus \text{check}(\text{e}(X, Y)) \iff \text{true}.
\]
\[
\text{e}(X, Y) \setminus \text{check}(\text{e}(X, Y)) \iff \text{p}(X, Y).
\]

Example (Marital status in CHR)

\[
\text{person}(X) \implies \text{check}(\text{person}(X)).
\]
\[
\text{married}(X) \setminus \text{check}(\text{person}(X)) \iff \text{true}.
\]
\[
\text{person}(X) \setminus \text{check}(\text{person}(X)) \iff \text{single}(X).
\]
CHR rules with special-case negation-as-absence

Assume negative part holds, otherwise repair later (changes semantics)

- Use \texttt{RHS} directly instead of auxiliary check
- Works if \texttt{RHS} nonempty, no built-ins, contains head variables

Definition (Rule scheme for CHR rule with negation in head)

CHR generalised simpagation rule

\[ N @ LHS1 \setminus LHS2 (\text{NEG1,NEG2}) \iff LHS3 | RHS \]

translates to CHR rules

\[ N2 @ \text{NEG1} \setminus \text{RHS} \iff \text{NEG2} | \text{true} \]
\[ N3 @ LHS1 \land \text{RHS} \setminus LHS2 \iff \text{true} \]
\[ N1 @ LHS1 \land LHS2 \Rightarrow LHS3 | \text{RHS} \]

- If \texttt{LHS2} is empty, rule \texttt{N3} can be dropped
Consequences and examples

- Shorter, more concise programs, often incremental, concurrent, declarative $\Rightarrow$ easier analysis
- Negation often not needed (if we have propagation rules)

**Example (Minimum in CHR)**

```prolog
num(Y) \ min(X) <=> Y<X | true.
num(X) ==> min(X).
```

**Example (Transitive closure in CHR, init-path rule)**

```prolog
p(X,Y) \ p(X,Y) <=> true.
e(X,Y) ==> p(X,Y).
```

**Example (Marital Status in CHR)**

```prolog
married(X) \ single(X) <=> true.
person(X) ==> single(X).
```
Simple conflict resolution (I)

Choose rule to be applied among applicable rules.
Assume (total) order for comparing rules.
Implementable for arbitrary CHR rules under refined semantics.

Definition (Rule scheme for CHR rule with static or dynamic weight)

Generalised simpagation rule (with weight, priority or probability $P$)

$$H_1 \setminus H_2 \iff \text{Guard} \mid \text{Body} : P$$

translates to CHR rules

$$H_1 \land H_2 \land \text{delay} \Rightarrow \text{Guard} \mid \text{rule}(P,H_1,H_2)$$
$$H_1 \setminus H_2 \land \text{rule}(P,H_1,H_2) \land \text{delay} \land \text{apply} \iff \text{Body} \land \text{delay} \land \text{apply}$$

- **delay**: auxiliary constraint to find applicable rules
- **rule**: contains an applicable rule
- **apply**: auxiliary constraint executes chosen rule
Simple conflict resolution (II)

One additional generic rule for rule choice

**Rule to resolve conflict**

\[
\text{choose @} \text{rule}(P_1,\_,\_) \setminus \text{rule}(P_2,\_,\_) \Leftrightarrow P_1 \geq P_2 \ | \ true
\]

Phase constraints `delay ∧ apply` present (at end of query):

- **Constraint** `delay` stores applicable rules in `rule`
- **Rule** `choose` selects rule with largest weight
- **Constraint** `apply` removes `delay` and executes chosen rule
- **Then** `delay` is called again
- **Then** `apply` is called again
Incremental general conflict resolution (I)

Choose rule to be applied among applicable rules. Implementable for arbitrary CHR rules under refined semantics.

Definition (Rule scheme for CHR rule with given property)

Generalised simpagation rule (with property $P$)

\[ H_1 \setminus H_2 \iff \text{Guard} \mid \text{Body} : P \]

translates to CHR rules

- **Rule delay**: finds applicable rules
- **Constraint conflictset**: collects applicable rules
- **Rule apply**: executes chosen rule
Incremental general conflict resolution (II)

Additional generic rules for rule choice

Rules to resolve conflict

\[
\text{collect @ conflictset}(L1) \land \text{conflictset}(L2) \iff \\
\text{append}(L1, L2, L3) \land \text{conflictset}(L3)
\]

\[
\text{choose @ fire} \land \text{conflictset}(L) \iff \\
\text{choose}(L, R, L1) \land \text{apply}(R) \land \text{conflictset}(L1) \land \text{fire}
\]

Phase constraint \text{fire present} (at end of query)

- **Rules** delay, collect collect applicable rules in conflictset
- **Constraint** fire present: rule choose selects rule R
  - Rule R applied by rule apply
  - Updated conflictset without applied rule added
  - Then fire is called again
Summary production rule systems in CHR

- Negation-as-absence and conflict resolution use very similar translation scheme
- Propagation and simpagation rules come handy
- Special case of negation-as-absence avoids negation at all
- Phase constraint avoids rule firing before conflict resolution
- Phase constraints relies on left-to-right evaluation order of queries
- Program sizes are roughly proportional to each other
- CHR complexity roughly as original production rule program
Event Condition Action rules

Extension of production rules for databases, generalise features like integrity constraints, triggers and view maintenance.

**ECA rules**

```plaintext
on Event if Condition then Action
```

- **Event**
  - triggers rules
  - external or internal
  - composed with logical operators and sequentially in time

- **(Pre-)condition**
  - includes database queries
  - satisfied if result non-empty

- **Action**
  - include database operations, rollbacks, IO and application calls
Issues in ECA rules

Technical and semantical questions arise

- Different results depending on point of execution.
  Solution: Coupling modes: immediately, later in the same or outside the transaction

- Applied to single tuples or sets of tuples?

- Application order of rules (priorities)

- Concurrent or sequential execution?

- Conflict resolution may be necessary

We choose solution that goes well with CHR
Embedding ECA rules in CHR

- Model events and database tuples as CHR constraints
- Update event constraints \texttt{insert/1, delete/1, update/2}

### Definition (Rule scheme for database relation)

\(n\)-ary relation \(r\) generates CHR rules

- \texttt{ins @ insert(R) } \Rightarrow R
- \texttt{del @ delete(P) \ R } \Leftrightarrow \text{match}(P,R) \mid \text{true}
- \texttt{upd @ update(R,R1) \ R } \Leftrightarrow R1

\((R= r(x_1,\ldots,x_n), R1= r(y_1,\ldots,y_n), x_i, y_j \text{ distinct variables})\)

\text{match}(P,R) \text{ holds if tuple } R \text{ matches tuple pattern } P

Additional generic rules to remove events (at end of program)

### Definition (Database operation event removal)

\texttt{insert(_)} \Leftrightarrow \text{true} \quad \texttt{delete(_)} \Leftrightarrow \text{true} \quad \texttt{update(_,_)} \Leftrightarrow \text{true}
Example (Salary increase)

Limit employee’s salary increase by 10 %

- **Before** update happens (by rule \texttt{upd})

Example

\[
\text{update}(\text{emp}(\text{Name}, S1) , \text{emp}(\text{Name}, S2)) \iff S2 > S1 \times (1+0.1) \ |
\text{update}(\text{emp}(\text{Name}, S1), \text{emp}(\text{Name}, S1 \times 1.1)).
\]

- **After** update happens (by rule \texttt{upd})

Example

\[
\text{update}(\text{emp}(\text{Name}, S1) , \text{emp}(\text{Name}, S2)) \iff S2 > S1 \times (1+0.1) \ |
\text{update}(\text{emp}(\text{Name}, S2), \text{emp}(\text{Name}, S1 \times 1.1)).
\]

- Difference: first argument of \texttt{update} in the body
More Examples

Production rule examples as ECA rules for database updates

Example (Transitive closure with ECA rules in CHR)

- `insert(p(X,Y)), p(X,Y) ==> delete(p(X,Y)).`
- `insert(e(X,Y)) ==> insert(p(X,Y)).`
- `insert(e(X,Y)), p(Y,Z) ==> insert(p(X,Z)).`
- `e(X,Y), insert(p(Y,Z)) ==> insert(p(X,Z)).`

Example (Marital Status with ECA rules in CHR)

- `insert(married(X)), single(X) ==> delete(single(X)).`
- `insert(person(X)) ==> insert(single(X)).`

Example (Minimum with ECA rules in CHR)

- `insert(num(Y)), min(X) ==> Y<X | delete(min(X)).`
- `num(Y), insert(min(X)) ==> Y<X | delete(min(X)).`
- `insert(num(X)) ==> insert(min(X)).`
LA formalism

- Hypothetical bottom-up logic programming language
- Features deletion of atoms and rule priorities
- Declarative production rule language, deductive database language, inference rules with deletion
- Designed to derive tight complexity results
- **The only implementation is in CHR**
- *It achieves the theoretically postulated complexity results!*
Logical Algorithm rules

**Definition (LA rules)**

\[ r @ p : A \rightarrow C \]

- **r**: rule name
- **p**: priority
  - arithmetic expression (variables must appear in first atom of \( A \))
  - either dynamic (contains variables) or static
- **A**: conjunction of user-defined atoms and comparisons
- **C**: conjunction of user-defined atoms (variables must appear in \( A \), i.e. range-restrictedness)
- **\( del(A) \)**: Deletion ("Negation") of positive atom \( A \), overshadows \( A \)
Logical Algorithm semantics

**Definition (LA semantics)**

- **LA state**: set of user-defined atoms
  - atoms occur positive, deleted (negative), or in both ways
- **LA initial state**: ground state
- **Rule applicable** to state if
  - lhs. atoms match state such that positive lhs. atoms do not occur deleted in state
  - lhs. comparisons hold under this matching
  - rhs. not contained in state (set-based semantics)
  - No other applicable rule with lower priority
- **LA final state**: no more rule applicable

Deletion by adding deletion atom \texttt{del}, no removal of atoms
Logical Algorithms in CHR

- Basically positive ground range-restricted CHR propagation rules
- Differences to CHR:
  - set-based semantics
  - explicit deletion atoms
  - redundancy test for rules to avoid trivial nontermination
  - rule priorities
Embedding LA in CHR

Definition (Rule scheme for LA predicate)

\( n \)-ary LA predicate \( a \) generates simpagation rules

\( A = a(x_1, \ldots, x_n) \) with \( x_i \) distinct variables)

\[ A \setminus A \iff \text{true}. \]
\[ \text{del}(A) \setminus \text{del}(A) \iff \text{true}. \]
\[ \text{del}(A) \setminus A \iff \text{true}. \]

Definition (Rule scheme for LA rule)

LA rule \( r @ p : A \rightarrow C \) translates to CHR propagation rule with priority

\[ r @ A_1 \Rightarrow A_2 | C : p \]

\((A1: \text{atoms from } A, A2: \text{comparisons from } A)\)

Priorities by CHR extension or conflict resolution
Ensuring set-based semantics

Applies to CHR rules in general (written as simplification rules)

- Generation of new rule variants by unifying head constraints

**Definition (Rule scheme for set-based semantics)**

To CHR simplification rules

\[ H \land H_1 \land H_2 \iff G \mid B[\land H_1 \land H_2] \]

add rules (if guard does not imply that head is body)

\[ H \land H_1 \iff H_1=H_2 \land G \mid B[\land H_1] \]

**Example**

\[ a(1,Y), \ a(X,2) \implies b(X,Y). \]

Additional rule from unifying \( a(1,Y) \) and \( a(X,2) \)

\[ a(1,2) \implies b(1,2). \]
LA example (Dijkstra’s shortest paths)

Example (Dijkstra in LA)

\[
d1 @ 1: \text{source}(X) \rightarrow \text{dist}(X,0)
d2 @ 1: \text{dist}(X,N) \land \text{dist}(X,M) \land N<M \rightarrow \text{del}(\text{dist}(X,M))
dn @ N+2: \text{dist}(X,N) \land \text{edge}(X,Y,M) \rightarrow \text{dist}(Y,N+M)
\]

Example (Dijkstra in CHR)

\[
\text{dist}(X,N) \setminus \text{dist}(X,N) \leftrightarrow \text{true}.
\text{del}(\text{dist}(X,N)) \setminus \text{del}(\text{dist}(X,N)) \leftrightarrow \text{true}.
\text{del}(\text{dist}(X,N)) \setminus \text{dist}(X,N) \leftrightarrow \text{true}.
\]

\[
d1 @ \text{source}(X) \Rightarrow \text{dist}(X,0) :1.
d2 @ \text{dist}(X,N), \text{dist}(X,M) \Rightarrow N<M \mid \text{del}(\text{dist}(X,M)) :1.
dn @ \text{dist}(X,N), \text{edge}(X,Y,M) \Rightarrow \text{dist}(Y,N+M) :N+2.
\]

Set-based transformation does not introduce more rules
Deductive database languages (*)

**Datalog**

- Inference rules with negation similar to Prolog
- Also related to database language SQL
- Negation as in production rule systems and Prolog
  - Only finite number of constants, no function symbols
  - Variables restricted to finite domains of constants
  - Rules are range-restricted
  - Stratification: No recursion through negation
- Evaluated bottom-up and set-based like Logical Algorithms
- For efficiency, evaluation is made query-driven
  - Magic-Set query transformation adds filter predicates to rules depending on input/output mode of query arguments
- **Not** turing-complete (finite data)
Embedding Datalog in CHR

Definition (Rule scheme for Datalog atom and Datalog rule)

\( n \)-ary Datalog atom \( a \) generates simpagation rule
\( (A = a(x_1, \ldots, x_n) \text{ with } x_i \text{ distinct variables}) \)

\[
A \ \blacktriangleright \ A \Leftrightarrow true.
\]

Datalog rule \( C \leftarrow A \) translates to CHR propagation rule

\[
r @ A_1 \Rightarrow A_2 \parallel C
\]

\( (A_1: \text{user atoms from } A, A_2: \text{built-ins from } A, C: \text{single user atom}) \)

- Analogous to translation of Logical Algorithms to CHR
- For set-based semantics, also see Logical Algorithms.
- Magic Set transformation adds filter atoms to \( A \) in Datalog rule
- Stratified Negation by negation-as-absence or conflict resolution
Example Datalog in CHR

Example (Transitive closure in Datalog)

\[
\begin{align*}
p(X,Y) & : - e(X,Y). \\
p(X,Z) & : - e(X,Y), p(Y,Z).
\end{align*}
\]

Example (Transitive closure translated to CHR)

\[
\begin{align*}
e(X,Y) \setminus e(X,Y) & \iff \text{true}. \\
p(X,Y) \setminus p(X,Y) & \iff \text{true}. \\
e(X,Y) & \Rightarrow p(X,Y). \\
e(X,Y), p(Y,Z) & \Rightarrow p(X,Z).
\end{align*}
\]

Set-based rule transformation does not introduce more rules
Constraint-based and logic-based programming

These are rule-based programming languages

- with logical variables subject to built-ins (like CHR)
- but no guards (except concurrent constraint languages)
- but no propagation rules (except deductive databases)
- but no multiple head atoms
- but with negation-as-failure and disjunction for search (in Prolog and constraint logic programming)
Prolog and Constraint Logic Programming

- Constraint logic programming (CLP) combines declarativity of logic programming and efficiency of constraint solving
- Prolog as CLP with syntactic equality as only built-in constraint
- Don’t-know nondeterminism by choice of rule (or disjunct)
- (Don’t-care nondeterminism by nonlogical cut operator)
- (Nonlogical Negation-as-failure)

**Definition (CLP program)**

CLP program: set of Horn clauses $A \leftarrow G$
(A atom, $G$ conjunction of atoms and built-ins)
CHR with disjunction – $\text{CHR}^\lor$

- CHR with disjunction in body ($\text{CHR}^\lor$): declarative formulation and clear distinction between don’t-know and don’t-care nondeterminism.
- Horn clause (CLP) program translates to equivalent $\text{CHR}^\lor$ program.
- CLP head unification and clause choice moved to body of $\text{CHR}^\lor$ rule.
- Required transformation is Clark’s completion.
Clark’s completion

Definition (Rule scheme for Clark’s completion for CLP clauses)

Clark’s completion of predicate $p/n$ defined by $m$ clauses

$$\bigwedge_{i=1}^{m} \forall (p(\bar{t}_i) \leftarrow G_i)$$

is the first-order logic formula

$$p(\bar{x}) \leftrightarrow \bigvee_{i=1}^{m} \exists \bar{y}_i \ (\bar{t}_i = \bar{x} \land G_i)$$

($\bar{t}_i$ sequences of $n$ terms, $\bar{y}_i$ variables in $G_i$ and $t_i, \bar{x}$ sequence of $n$ new variables)
CLP translation to CHR

For pure Prolog and CLP without cut and negation-as-failure

Definition (Rule scheme for pure (C)LP clauses)
- CLP predicate $p/n$ is considered as CHR constraint
- For each predicate $p/n$ Clark’s completion of $p/n$ added as CHR simplification rule

Example (Append in Prolog)

```prolog
append([], L, L) ← true.
append([H|L1], L2, [H|L3]) ← append(L1, L2, L3).
```

Example (Append in CHR)

```prolog
append(X, Y, Z) ⇔
    ( X=[] ∧ Y=L ∧ Z=L
    ∨ X=[H|L1] ∧ Y=L2 ∧ Z=[H|L3] ∧ append(L1, L2, L3) ).
```
Example - Prime sieve programs

Comparison between Prolog and CHR by example program

Example (Prime sieve in Prolog)

```prolog
primes(N,Ps):- upto(2,N,Ns), sift(Ns,Ps).

upto(F,T,[]):- F>T, !.
upto(F,T,[F|Ns1]):- F1 is F+1, upto(F1,T,Ns1).

sift([],[]).
sift([P|Ns],[P|Ps1]):- filter(Ns,P,Ns1), sift(Ns1,Ps1).

filter([],P,[]).
filter([X|In],P,Out):- X mod P =:= 0, !, filter(In,P,Out).
filter([X|In],P,[X|Out1]):- filter(In,P,Out1).
```

Prolog uses nonlogical cut operator.

Example (Prime sieve in CHR)

```
upto(N) <=> N>1 | M is N-1, upto(M), prime(N).
sift @ prime(I) \ prime(J) <=> J mod I =:= 0 | true.
```
Example - Shortest path program

Comparison between Prolog and CHR by example program

**Example (Shortest path in Prolog)**

```prolog
p(From, To, Path, 1) :- e(From, To).
p(From, To, Path, N) :- e(From, Via),
    not member(Via, Path),
    p(Via, To, [Via|Path], N1),
    N is N1+1.
shortestp(From, To, N) :- p(From, To, [], N),
    not (p(From, To, [], N1), N1<N).
```

Prolog uses nonlogical negation-as-failure.

**Example (Shortest path in CHR)**

```prolog
p(X, Y, N) \ p(X, Y, M) <=> N=<M | true.
e(X, Y) ==> p(X, Y, 1).
e(X, Y), p(Y, Z, N) ==> M is N+1, p(X, Z, M).
```
Concurrent constraint programming

- Concurrent constraint (CC) language framework
  - Permits both nondeterminisms
  - One of the frameworks closest to CHR
  - We concentrate on the committed-choice fragment of CC
    (Based on don’t-care nondeterminism like CHR)

**Definition (Abstract syntax of CC program)**

CC program is a finite sequence of declarations that define agent.

\[
\begin{align*}
\text{Declarations} & \quad D ::= \quad p(\tilde{t}) \leftarrow A \mid D, D \\
\text{Agents} & \quad A ::= \quad true \mid c \mid \sum_{i=1}^{n} c_i \rightarrow A_i \mid A || A \mid p(\tilde{t})
\end{align*}
\]

(\(p\) user-defined predicate symbol, \(\tilde{t}\) sequence of terms, \(c\) and \(c_i\)’s constraints)
Ask-and-tell

- **Ask-and-tell**: communication mechanism of CC (and CHR)
- **Tell**: Add a constraint to the constraint store (producer / server)
- **Ask**: Inquiry whether or not constraint holds (consumer / client)
  - Realized by logical entailment
  - Checks whether constraint is implied by constraint store
- Generalizes idea of concurrent data flow computations
  - Operation waits until its parameters are known
States are pairs of agents and built-in constraint store

**Definition (Ask and Tell)**

**Tell:** adds constraint $c$ to constraint store

$$\langle c, d \rangle \rightarrow \langle \text{true}, c \land d \rangle$$

**Ask:** nondeterministically choose constraint $c_i$ (implied by $d$) and continue with agent $A_i$

$$\langle \sum_{i=1}^{n} c_i \rightarrow A_i, d \rangle \rightarrow \langle A_j, d \rangle \quad \text{if} \quad CT \models \forall (d \rightarrow c_j) \quad (1 \leq j \leq n)$$
CC operational semantics (II)

Definition (Composition and Unfold)

**Composition**: Operator $\parallel$ defines concurrent composition of agents

$$
\langle A, c \rangle \rightarrow \langle A', c' \rangle
$$

$$
\langle (A \parallel B), c \rangle \rightarrow \langle (A' \parallel B), c' \rangle
$$

$$
\langle (B \parallel A), c \rangle \rightarrow \langle (B \parallel A'), c' \rangle
$$

**Unfold**: replaces agent $p(\tilde{t})$ by its definition

$$
\langle p(\tilde{t}), c \rangle \rightarrow \langle A, \tilde{t} = \tilde{s} \land c \rangle \quad \text{if} \quad (p(\tilde{s}) \leftarrow A) \text{ in program } P
$$
Embedding in CHR

- CC predicates → CHR constraints
- CC constraints → CHR built-in constraints
- CC declaration → CHR simplification rule
- CC agent → CHR goal
- CC ask expression → CHR simplification rules for auxiliary unary
  CHR constraint ask
- Ask constraint → built-in in guard of CHR rule
- Tell constraint → built-in in body of CHR rule
Translation

Definition (Rule scheme for CC expressions)

Declarations and agents are translated from CC

\[
D ::= p(\bar{t}) \leftarrow A \mid D, D
\]
\[
A ::= true \mid c \mid \sum_{i=1}^{n} c_{i} \rightarrow A_{i} \mid A \land A \mid p(\bar{t})
\]

to CHR as

\[
D^{CHR} ::= p(\bar{t}) \Leftrightarrow A \mid D, D
\]
\[
A^{CHR} ::= true \mid c \mid \text{ask}(\sum_{i=1}^{n} c_{i} \rightarrow A_{i}) \mid A \land A \mid p(\bar{t})
\]

For each CC Ask \( A \) of the form \( \sum_{i=1}^{n} c_{i} \rightarrow A_{i} \) also generate \( n \) single-headed simplification rules for unary \text{ask} constraint

\[
\text{ask}(A) \Leftrightarrow c_{i} \mid A_{i} (1 \leq i \leq n).
\]
Example (Maximum)

Example (Maximum in CC)

\[
\text{max}(X, Y, Z) \leftarrow (X \leq Y \rightarrow Y = Z) + (Y \leq X \rightarrow X = Z).
\]

Example (Maximum in CHR)

\[
\text{max}(X, Y, Z) \iff \text{ask}((X \leq Y \rightarrow Y = Z) + (Y \leq X \rightarrow X = Z)).
\]

\[
\text{ask}((X \leq Y \rightarrow Y = Z) + (Y \leq X \rightarrow X = Z)) \iff X \leq Y \mid Y = Z.
\]

\[
\text{ask}((X \leq Y \rightarrow Y = Z) + (Y \leq X \rightarrow X = Z)) \iff Y \leq X \mid X = Z.
\]

To simplify rules replace \( \text{ask}((X \leq Y \rightarrow Y = Z) + (Y \leq X \rightarrow X = Z)) \) by \( \text{ask}_{\text{max}}(X, Y, Z) \)

Example (Simplified maximum in CHR)

\[
\text{ask}_{\text{max}}(X, Y, Z) \iff X \leq Y \mid Y = Z.
\]

\[
\text{ask}_{\text{max}}(X, Y, Z) \iff Y \leq X \mid X = Z.
\]
Embeddings in CHR

Advantages

- Advantages of CHR for **execution**
  - Efficiency, also optimal complexity possible
  - Abstract execution by constraints, even when arguments unknown
  - Incremental, anytime, online algorithms for free
  - Concurrent, parallel for confluent programs

- Advantages of CHR for **analysis**
  - Decidable confluence and operational equivalence
  - Estimating complexity semi-automatically
  - Logic-based declarative semantics for correctness

- **Embedding allows for comparison and cross-fertilization (transfer of ideas)**
Potential shortcomings of embeddings in CHR

⇒ Use extensions of CHR (dynamic CHR covers all
  ➤ for built-in “negation” of rb systems, deductive db and Prolog
    ⇒ CHR with negation-as-absence
  ➤ for conflict resolution of rule-based systems
    ⇒ CHR with priorities
  ➤ for built-in search of Prolog, constraint logic programming
    ⇒ CHR with disjunction or search library
  ➤ for ignorance of duplicates of rule-based formalisms
    ⇒ CHR with set-based semantics
  ➤ for diagrammatic notation of graph-based systems
    ⇒ CHR with graphical interface

Instead of extensions, special-purpose CHR programs can be used.
Positive ground range-restricted CHR

- All approaches can be embedded into simple CHR fragment (except Prolog, constraint logic programming)
  - **ground**: queries ground
  - **positive**: no built-ins in body of rule
  - **range-restricted**: variables in guard and body also in head

- These conditions imply
  - Every state in a computation is ground
  - CHR constraints do not delay and wake up
  - Guard entailment check is just test
  - Computations cannot fail

- Conditions can be relaxed: auxiliary functions as non-failing built-ins in body
Distinguishing features of CHR for programming

*Unique combination of features*

- **Multiple Head Atoms** not in other programming languages
- **Propagation rules** only in production rules, deductive databases, Logical Algorithms
- **Constraints** only in constraint-based programming
  - Logical variables instead of ground representation
  - Constraints are reconsidered when new information arrives
  - Notion of failure due to built-in constraints
- **Logical Declarative Semantics** only in logic-based prog.
  - CHR computations justified by logic reading of program
Embedding fragments of CHR in other rule-based approaches

Possibilities are rather limited (without interpreter or compiler)

- **Positive ground range-restricted fragment** embeddable into
  - Rule-based systems with negation and Logical Algorithms
  - Only simplification rules in Rewriting- and Graph-based approaches (except Petri-nets)
  - Only propagation rules in deductive databases

- **Single-headed rules** embeddable into
  - Concurrent constraint programming languages
The Potential of Constraint Handling Rules

CHR - an essential unifying computational formalism?
Rule-based Systems, Formalisms and Languages can be compared and cross-fertilize each other via CHR!

- CHR is a logic and a programming language
- CHR can express any algorithm with optimal complexity
- CHR is efficient and extremely fast
- CHR supports reasoning and program analysis
- CHR programs are anytime, online and concurrent algorithms
- CHR has many applications from academia to industry

The first formalism and the first language for students
Reasoning formalism and programming language for research
CHR - a Lingua Franca for computer science!