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## Constraint Handling Rules - Getting started

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### Getting started

How CHR works

CHR programs and their execution

## Overview

- ▶ Basic introduction to CHR using examples
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- ▶ Informal description of rule execution in CHR

## CHR implementations

- ▶ Most recent and advanced implementation: K.U. Leuven (recommended)
- ▶ Programs also executable with minor changes in other Prolog implementations of CHR
- ▶ K.U. Leuven JCHR: CHR implementation in Java
- ▶ K.U. Leuven CHR library for C
- ▶ CHR code (declarations and rules) and host language statements mixed in programs

## Declarations

Declarations introduce CHR constraints we will define by rules

### Example (Declarations)

```
:- module(weather, [rain/0]).  
:- use_module(library(chr)).  
:- chr_constraint rain/0, wet/0, umbrella/0.
```

- ▶ Functor notation  $c/n$ : name  $c$ , number of arguments  $n$  of constraint  $c(t_1, \dots, t_n)$
- ▶ First line: optional Prolog module declaration: declares module `weather`, where only constraint `rain/0` is exported.
- ▶ Second line: loading CHR library
- ▶ Third line: Defines CHR constraints `rain`, `wet`, and `umbrella`
  - ▶ At least name and arity must be given

## Rules (I)

- ▶ Parts of a rule:
  - ▶ Optional name
  - ▶ Left-hand side (l.h.s.) called head, with optional guard
  - ▶ Right-hand side (r.h.s) called body
- ▶ Head, guard, and body consist of constraints
- ▶ Three different kind of rules

## Rules (II)

### Example (Rules)

```
rain ==> wet.
```

```
rain ==> umbrella.
```

- ▶ First rule: “If it rains, then it is wet”
- ▶ Second rule: “If it rains, we need an umbrella”
- ▶ Head of both rules is `rain`
- ▶ Bodies: `wet` and `umbrella`
- ▶ No guards
- ▶ Also called propagation rules (`==>`)
  - ▶ Do not remove constraints, only add new ones

## Queries

- ▶ Posing query initiates computations
- ▶ Rules applied to query until exhaustion (no more changes happen)
- ▶ Rule applications manipulate query by removing and adding constraints
- ▶ Result (called answer) consists of remaining constraints

### Example (Query)

```
rain ==> wet.
```

```
rain ==> umbrella.
```

Posing query `rain` results in `rain, wet, umbrella`  
(not necessarily in this order)



## Top-down execution

- ▶ Rules applied in textual order
- ▶ In general: If more than one rule applicable, one rule is chosen
- ▶ Rule applications cannot be undone like in Prolog  
⇒ CHR is a committed-choice language

### Example (Top-down execution)

Two simplification rules

```
rain <=> wet.
```

```
rain <=> umbrella.
```

- ▶ Application of first rule removes `rain`
- ▶ Second rule never applied

## Simplification rules

- ▶ Propagation rules
  - ▶ Drawing conclusions from existing information
- ▶ Simplification rules
  - ▶ Simplify things
  - ▶ Express state change
  - ▶ Dynamic behavior

## Example

### Example (Walk)

- ▶ Walk expressed by movements `east, west, south, north`
- ▶ Multiplicity of steps matters, order does not matter for walk
- ▶ Simplification rules express that steps can cancel out each other (i.e. `east and west`)

`east, west <=> true.`

`south, north <=> true.`

- ▶ Rules simplify walk to one with minimal number of steps
- ▶ Query `east, south, west, west, south, south, north, east, east`  
yields answer `east, south, south`

## Logical variables

### Logical variables

- ▶ Featured in declarative languages like CHR
- ▶ Similar to mathematical unknowns and variables in logic
- ▶ Can be unbound or bound
- ▶ Bound variables indistinguishable from value they are bound to
- ▶ Bound variables cannot be overridden
- ▶ Languages with such variables called single-assignment languages
- ▶ Other languages like C and Java feature destructive (multiple) assignments

## Example

### Example

- ▶ Two constraints with one argument representing men (e.g. `male(joe)`) and women (e.g. `female(sue)`)
- ▶ Assigning men and woman for dancing with simplification rule `male(X), female(Y) <=> pair(X,Y)`.
- ▶ Variables `X`, `Y` placeholders for values of constraints matching rule head
- ▶ Scope of variable is rule it appears in
- ▶ Given query with several men and women, rule pairs them until only people of one sex left

## Types of rules

### Example (Propagation rule)

- ▶ Computing all possible pairs with propagation rule (keeps `male` and `female` constraints)

```
male(X), female(Y) ==> pair(X,Y).
```

- ▶ Number of pairs quadratic in number of people  
⇒ Propagation rule can be expensive

### Example (Simpagation rule)

- ▶ One man dances with several women expressed by simpagation rule

```
male(X) \ female(Y) <=> pair(X,Y).
```

- ▶ Head constraints left of backslash `\` kept, head constraints right of backslash removed

## Example

### Example (Family relationships (I))

- ▶ Propagation rule named `mm` expresses grandmother relationship  
`mm @ mother(X,Y), mother(Y,Z) ==> grandmother(X,Z).`
- ▶ Constraint `grandmother(joe,sue)` reads as “Grandmother of Joe is Sue”
- ▶ Allows derivation of grandmother relationship from mother relationship
- ▶ `mother(joe,ann), mother(ann,sue)` will propagate `grandmother(joe,sue)` using rule `mm`

## Built-in constraints

- ▶ Two kinds of constraints in CHR
- ▶ CHR constraints (user-defined constraints)
  - ▶ Declared in current program and defined by CHR rules
- ▶ Built-in constraints (built-ins)
  - ▶ Predefined in host language or imported CHR constraints from other modules
- ▶ On left hand side CHR and built-ins constraints separated into head and guard
- ▶ On right hand side freely mixed



## Syntactic equality

### Example (Family relationships (II))

- ▶ Mother of a person is unique, expressed by rule

`dm @ mother(X, Y) \ mother(X, Z) <=> Y=Z.`

- ▶ Syntactic equality: Mother relation is function, first argument determines second
- ▶ Rule enforces this using built-in syntactic equality =
  - ▶ Constraint `Y=Z` makes sure that both variables have the same value
  - ▶ Occurrences of one variable are replaced by (value of) other variable
- ▶ Query `mother(joe, ann), mother(joe, ann)` will lead to `mother(joe, ann)`
  - ▶ `ann=ann` simplified away, is always *true*

## Failure

### Example (Family relationship (III))

```
dm @ mother(X,Y) \ mother(X,Z) <=> Y=Z.
```

- ▶ Query `mother(joe,ann), mother(joe,sue)` fails (Joe would have two different mothers)
  - ▶ Rule `dm` will lead to `ann=sue`, which cannot be satisfied
  - ▶ Built-in acts as test in this case
- 
- ▶ Failure aborts computation
  - ▶ Failure leads to answer `no` in most Prolog systems

## Variables in queries and head matching

- ▶ Query can contain variables (matching successful as long as they are not bound by matching)

### Example (Family relationship (IV))

```
mm @ mother(X,Y), mother(Y,Z) ==> grandmother(X,Z).
```

- ▶ Answer `grandmother(A,C)` for query `mother(A,B), mother(B,C)`
- ▶ No rule applicable to `mother(A,B), mother(C,D)`
- ▶ Answer `grandmother(A,D)` when built-in added to query:  
`mother(A,B), mother(C,D), B=C`
- ▶ Adding `A=D` instead leads to `grandmother(C,B)`
- ▶ Adding `A=C` makes rule `dm` applicable,  
leads to `mother(A,B), A=C, B=D`

## Example (I)

### Example (Mergers and acquisitions)

- ▶ CHR constraint `company (Name, Value)` represents company with market value `Value`
- ▶ Larger company buys company with smaller value expressed by rule

```
company (Name1, Value1), company (Name2, Value2) <=>  
    Value1 > Value2 | company (Name1, Value1 + Value2).
```

- ▶ Guard `Value1 > Value2` acts as precondition of rule applicability
- ▶ Only built-ins allowed in guard

## Example (II)

### Example (Mergers and acquisitions cont.)

- ▶ In line arithmetic expression `Value1+Value2` works for host language Java
- ▶ In Prolog `is` has to be used leading to rule

```
company (Name1, Value1), company (Name2, Value2) <=>  
    Value1>Value2 | Value is Value1+Value2,  
    company (Name1:Name2, Value) .
```

- ▶ Rule is applicable to any pair of companies with different value
- ▶ After exhaustive only a few companies will remain (all with the same value)

## Concrete Syntax

- ▶ CHR-specific part of program consists of declarations and rules
- ▶ Declarations are implementation-specific
- ▶ In following EBNF grammar:
  - ▶ Terminals in single quotes
  - ▶ Expressions in square brackets optional
  - ▶ Alternatives separated by |

## Rules

```
Rule --> [Name '@']
  (SimplificationRule | PropagationRule | SimpagationRule) '.'

SimplificationRule -->
  Head          '<=>' [Guard '|'] Body
PropagationRule  -->
  Head          '==>' [Guard '|'] Body
SimpagationRule  -->
  Head '\ ' Head '<=>' [Guard '|'] Body
```

- ▶ Three different types of rules in CHR
- ▶ ' | ' separates guard from body of rule
- ▶ ' \ ' separates head of simpagation rule into two parts

## Rules

```
Head          --> CHRConstraints
Guard         --> BuiltInConstraints
Body          --> Goal

CHRConstraints -->      CHRConstraint
                  | CHRConstraint ',' CHRConstraints
BuiltInConstraints --> BuiltIn
                  | BuiltIn ',' BuiltInConstraints
Goal           -->      CHRConstraint | BuiltIn | Goal ',' Goal

Query         --> Goal
```

- ▶ Head of rule is sequence of CHR constraints
- ▶ Guard is a sequence of built-ins constraints
- ▶ Body is a sequence of built-ins and CHR constraints



## Basic built-in constraints (I)

- ▶ Using set of predicates from host language Prolog
- ▶ Can be used for auxiliary computations in rule body
- ▶ Built-ins in guard of rule usually test (succeed or fail)
  
- ▶ Most basic built-ins
  - ▶ `true/0` always succeeds
  - ▶ `fail/0` never succeeds
  
- ▶ Testing if variables are bound
  - ▶ `var/1` tests if argument is unbound variable
  - ▶ `nonvar/1` tests if argument is bound variable

## Basic built-in constraints (II)

- ▶ Syntactical identity of expressions (infix):
  - ▶ `=/2` makes arguments syntactically identical by binding variables (fails if binding not possible)
  - ▶ `==/2` tests if arguments syntactically identical
  - ▶ `\=/2` tests if arguments syntactically different
- ▶ Computing and comparing arithmetic expressions (infix):
  - ▶ `is/2` binds first argument to value of arithmetic expression in the second argument (fails if not possible)
  - ▶ `</2, =</2, >/2, >=/2, =:/2, =\=/2` test if arguments are arithmetic expressions whose values satisfy comparison

## Basic built-in constraints (III)

- ▶ `=/2` and `is/2` bind first argument  
⇒ should never be used in guards
- ▶ Use `==/2` and `:=/2` instead
- ▶ But some compilers make silent replacement

## Informal semantics

- ▶ Description of current sequential implementation
- ▶ Based on so-called refined operational semantics
- ▶ Maybe different rule application in parallel, experimental and future implementations
- ▶ Those implementations will still respect so-called abstract operational semantics

## Constraints

- ▶ Constraint is active operation as well as passive data
- ▶ Constraints in goals processed from left to right
- ▶ When CHR constraint encountered:
  - ▶ Evaluated like procedure call
  - ▶ Checks applicability of rules it appears in
  - ▶ Called active constraint
- ▶ Rules applied in textual order
- ▶ If no rule applicable to active constraint it becomes passive and is put in constraint store
- ▶ Passive constraints become active again context changes (their variables get bound)

## Head matching

- ▶ One head constraint of rule is matched against active constraint
- ▶ Matching succeeds if constraint serves pattern
- ▶ Matching may bind variables in head (not in active constraint)
- ▶ If matching succeeds and rule head consists of more than one constraint, constraint store is searched for partner constraints to match other head constraints

- ▶ Head constraints searched from left to right
- ▶ Exception: simpagation rule
  - ▶ Constraints to be removed searched for before constraints to be kept are searched for
- ▶ If matching succeeds, guard is checked
- ▶ If several head constraints match active constraint, rule tried for each matching
- ▶ If no successful matching exists, active constraint tries next rule

## Guard checking

- ▶ Guard is precondition on rule applicability
- ▶ Test that either succeeds or fails
- ▶ If guard succeeds, rule is applied
- ▶ If guard fails, active constraint tries next head matching



## Body execution

- ▶ When rule is applied, we say it fires
- ▶ Simplification rule: matching constraints removed, body executed
- ▶ Simpagation rule: similar to simplification rule but constraints matching head part preceding \ kept.
- ▶ Propagation rule: Body executed without removing any constraints
- ▶ Propagation rule will not fire with same constraint again
- ▶ According to rule type head constraints either called kept or removed
- ▶ Next rule tried when active constraint not removed